An Interesting Diophantine Problem on Triples-III

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Abstract – This paper concerns with an interesting Diophantine problem and aims at determining explicitly three non-zero distinct integers a, b, c such that, $a+b+N, a+c+N, b+c+N$ and $a+b+c+N$ is a perfect square. A methods has been considered to obtain the three required integers a,b,c. This shows that there are many triples in integers, each satisfying the considered kind of pattern among its members.

Index Terms – Diophantine Problem, Integer Triple, System of Equations, 2010 Mathematics Subject Classification: 11D99, 11D09.

1. INTRODUCTION

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets we have studied by Diophantus [1]. A set of m positive integers $\{a_1, a_2, a_3, ...a_m\}$ is said to have the property D(n) , $n \in Z-\{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuple with property D(n).

Many mathematicians considered the construction of different formulations of Diophantine Triples with the property D(n) for any arbitrary integer n and also, for any linear polynomials in n. In this context, one may after [2-19] for an extensive review of various problems on Diophantine Triples. In [20-23], the construction of special Dio – Triples, special Dio – Quadruples are considered and the special mention is provided because it differs from the earlier one and the special Dio-Triple (special Dio-Quadruple) is constructed where the product of any two numbers of the triple (quadruple) with addition of the same member and the addition with a non-zero integer or a polynomial with integer co-efficients satisfies the required property.

This paper aims at constructing an interesting triple where, the sum of any two members of the set or their sum such that, if a non- zero integer is added to the sum of any pair of them as well as to their sum, the results are all squares.

2. METOHD OF ANALYSIS

Let N be any given non-zero integer. Let a, b, c be three non-zero distinct integers such that

$$a+b = x^2 + 2(N-1)x + (N-1)^2 - N \tag{1}$$

$$b+c = x^2 + 2(N-2)x + (N-2)^2 - N \tag{2}$$

$$a+b+c = x^2 + 2(N+5)x + (N+5)^2 - N \tag{3}$$

From (1) and (3), we get

$$c = 12x + 12N + 24 \tag{4}$$

From (2), note that

$$b = x^2 + 2Nx + N^2 - 8x - 9N - 20 \tag{5}$$

From (3) and (2), we get

$$a = 6x + 6N + 21 \tag{6}$$

It is noticed that each of the expressions $a+b+N, b+c+N, a+b+c+N$ is a perfect square.

Now, $a+c+N = 18x + 19N + 45 = y^2$ (say) (7)

Let (x_0, y_0) be any solution satisfying (7). Then, after a few calculations, it is found that the general values of x and y in terms of x_0, y_0 are given by [24]

$$x_n = x_0 + n^2 a - 2ny_0 \tag{8}$$

$$y_n = (-1)^n (y_0 - na)$$

Substituting the values of x in (4)-(6), we obtain the required triples. Note that on employing (8), many triples are generated and it is worth mentioning that the values of x for which a,b and c are distinct are to be considered. A few numerical examples of a,b and c for $N=5,7$ are presented in the Table 1 below:

N	y_0	n	x_0	a	b	c
5	$18k-11$	0	$18k^2-22k$	$108k^2-132k+45$	$(18k^2-22k)^2-40$	$216k^2-264k-72$
		1	$18k^2-58k+40$	$108k^2-348k+285$	$(18k^2-58k+40)^2$ -40	$216k^2-696k+552$
		2	$18k^2-94k+116$	$108k^2-564k+741$	$(18k^2-94k+116)^2$ -40	$216k^2-1128k+1464$
		3	$18k^2-130k+228$	$108k^2-780k+1413$	$(18k^2-130k+228)^2$ -40	$216k^2-1560k+2808$
5	$18k-7$	0	$18k^2+14k-4$	$108k^2+84k+21$	$(18k^2+14k-4)^2$ -40	$216k^2-168k+24$
		1	$18k^2-50k+28$	$108k^2-300k+213$	$(18k^2-50k-28)^2$ -40	$216k^2-600k+408$
		2	$18k^2-86k+96$	$108k^2-516k+621$	$(18k^2-86k+96)^2$ -40	$216k^2-1032k+1224$
		3	$18k^2-122k+200$	$108k^2-732k+1245$	$(18k^2-122k+200)^2$ -40	$216k^2-1464k+2472$
5	$18k-14$	0	$18k^2-28k+1$	$108k^2-168k+69$	$(18k^2-28k+1)^2$ $+108k^2-168k-28$	$216k^2-336k+120$
		1	$18k^2-64k+47$	$108k^2-384k+345$	$(18k^2-64k+47)^2$ $108k^2-384k+248$	$216k^2-768k+672$
		2	$18k^2-100k+129$	$108k^2-600k+837$	$(18k^2-100k+129)^2$ $+108k^2-600k+740$	$216k^2-1200k+1656$
		3	$18k^2-136k+247$	$108k^2-816k+1545$	$(18k^2-136k+247)^2$ $108k^2--816k+1448$	$216k^2-1632k+3072$
7	$18k-4$	0	$18k^2-8k-9$	$108k^2-48k+9$	$(18k^2-8k-9)^2$ $+108k^2-48k+9$	$216k^2-96k$

		1	$18k^2-44k+17$	$108k^2-264k+165$	$(18k^2-44k+17)^2 +108k^2-264k+68$	$216k^2-528k+312$
		2	$18k^2-80k+79$	$108k^2-480k+537$	$(18k^2-80k+79)^2 +108k^2-480k+440$	$216k^2-960k+1056$
		3	$18k^2-116k+177$	$108k^2-696k+1125$	$(18k^2-116k+177)^2 +108k^2-696k+1028$	$216k^2-1392k+2232$

Table 1 Numerical Examples

3. CONCLUSION

In this paper, we have considered an interesting Diophantine problem of constructing triples, which are such that, in each triple, the sum of any two as well as their sum, when added with a non-zero integer, represents a perfect square. The beauty of many Diophantine problems lies in the fact that they are neither trivial nor difficult to analyze. To conclude, one may investigate several further add new explicit Diophantine problems.

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REFERENCES

[1] I.G. Bashmakova (ed.), Diophantus of Alexandria, Arithmetics and The Book of Polygonal Numbers, Nauka , Moskow,1974.
[2] A.F. Beardon and M.N. Deshpande, Diophantine triples, The Mathematical Gazette 86, 2002, 258-260.
[3] Bo He, A. Togbe , On a family of Diophantine triples $\{k+1,4k,9k+3\}$, Period Math Hungar, 58, 2009, 59-70.
[4] Bo He, A. Togbe , On a family of Diophantine triples $\{k+1, A^2k+2A, (A+1)^2k+2(A+1)\}$ with two parameters, Acta Math Hunger, 124,2009,99-113.
[5] Bo He, A. Togbe , On a family of Diophantine triples with two parameters, Period Math Hunger, 64,2012,1-10
[6] Y.Begeaud ,A.Dujella and M.Mignotte , On the family of Diophantine triples $\{k-1, k+1, 16k^3-4k\}$, Glasgow Math.J, 49, 2007,333-334.
[7] M.N.Deshpande , and E.Brown, Diophantine triples and the pell sequence, Fibonacci, Quart,39,2001,242-249.
[8] M.N. Deshpande , On interesting family of Diophantine triples, internat. J.Math.Ed. Sci.Tech.,33, 2002, 253-256.
[9] M.N.Deshpande, Family of Diophantine triples , Bulletin of the Marathwada Mathematical Society,4,2003, 19-21.
[10] A.Dujella and C.Fuchs, Complete solution of the polynomial version of a problem Diophantus, J. Number theory 106, 2004,326-344.
[11] A.Dujella and F.Luca, On a problem of Diophantus With Polynomials, Rocky Mountain J. math, 37,2007,131-157.
[12] A.Dujella and V.Petricevic, strong Diophantine triples, Experiment Math 17, 2008, 83-89.
[13] A.Fillipin, Bo He, A.Togbe, On the D(4)-triple $\{F_{2k}, F_{2k+6}, 4F_{2k+4}\}$, Fibonacci Quart ,48,2012,219-227.
[14] A.Fillipin, Bo He, A.Togbe, On a family of two parameters D(4)-triples, Glas. Mat. Ser.III,47,201231-51.
[15] A.Fillipin, Non - extend ability of D(-1) triples of the form{1,10,c}, interna. J . Math. Math.sci.,35,2005,2217-2226.
[16] M.A.Gopalan, and G.Srividhya, Two special Diophantine triples, Diophantus J.Math.,1(1), 2012, 23-27.
[17] M.A.Gopalan , V.Sangeetha, Manju somnath, Construction of the Diophantine Triple involving polygonal numbers, Sch.J.Eng.Tech.,2(1),2014,19-22.
[18] M.A.Gopalan, S.Vidhyalakshmi, S.Mallika, Special family of Diophantine triples, Sch. J. Eng. Tech,. 2(2A),2014, 197-199.
[19] V.Pandichelvi, Construction of the Diophantine Triple Involving Polygonal Numbers, Impact J. Sci.Tech., Vol.5., No.1, 2011, 07-11 .
[20] M.A.Gopalan, G.Sumathi, S.Vidhyalakshmi, Dio -Quadruple involving Jacobsthal Lucas Number and Thabit-Ibn-Kurrah Number with property D(1), IJIRR, Vol 2(2),2014,47-50.
[21] M.A.Gopalan, G.Sumathi,S.Vidhyalakshmi, Special Dio-Quadruple involving Jacobsthal and Jacobsthal Lucas Numbers with property D (k^2+1) . IJMSEA , Vol 8 No:III,2014,221-225
[22] K.Meena, S.Vidhyalakshmi, M.A.Gopalan, R.Presenna, Special Dio-Triples, JP Journal of Algebra, NT and Applications Vol. 34, No.1, 13-25,2014.
[23] K.Meena, S.Vidhyalakshmi, M.A.Gopalan, R.Presenna, Sequences of Special Dio-Triples, IJMTT, Vol. 10, No.1, 43-46, 2014.
[24] M.A.Gopalan, R.Ganapathy, A Remark on Linear Functions Made Squares, Acta Ciencia Indica , Vol-XXV11M, No 4, 557, 2001.